GOX / Paraffin Hybrid Motor - Nozzle Design

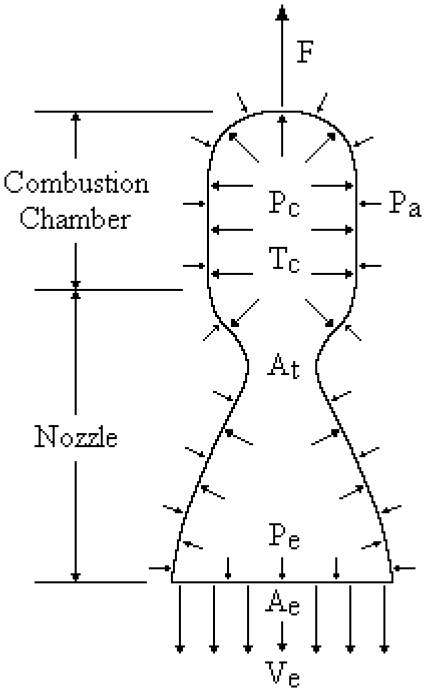
The following is an analysis for the design of a graphite nozzle to be use in the gox/paraffin hybrid motor. The analysis borrows heavily from the overview on nozzle theory done by Richard Nakka [1] and Benito Loyala [2]. The formula derivations were done with the help of Cengels Thermodynamics text [3] and Munsons Fluid Mechanics text [4]. Also used was PROPEP software package [5] to determine gox/paraffin propellant properties.

Theory:

The force produced by a rocket motor can be described as being equivalent to the pressure acting over the motor's chamber and nozzle:

$$\text{Force} := \int \text{Pressure}\, d\text{Area}$$

This is illustrated in the following figure:



The force can be expressed as the sum of the force produced by expelling the mass of the exhaust products out at a specified rate and velocity plus the 'pressure thrust' created by the difference in the exit pressure and the ambient pressure applied to the end of the rocket motor.

$$Thrust := \int Pressure\ dArea$$

$$Thrust := ForceMassExpelled + ForcePressureThrust$$

$$Thrust := Mdot \cdot Ve + (Pe - Pa) \cdot Ae$$

Where :
Mdot = Mass flow rate of exhaust products.
Ve = Exhaust velocity.
Pe = Nozzle exit pressure
Pa = Ambient pressure
Ae = Nozzle exit area

If we assume that the nozzle throat is choked and that conservation of mass holds true then the amount of mass passing through the nozzle throat area at a speed of Mach 1 (choked flow) is going to equal the amount of fuel and oxidizer being consumed:

Mdot = ρ*A*V*

Where :
ρ* = ρstar = Critical exhaust products density
A* = Astar = Critical throat area
V* = Vstar = Critical throat velocity (Ma=1)

Therefore we can reexpress the thrust of our motor as:

Thrust = ρ*A*V*Ve + (Pe - Pa)*Ae

Using the magical equations for:

1. Energy
2. Continuity
3. Momentum
4. Equation of state

This section to be expanded upon

We achieve the following equation:

$$Thrust := Astar \cdot Pc \cdot \sqrt{\frac{2 \cdot k^2}{k-1}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}} \cdot \left[1 - \left(\frac{Pe}{Pc}\right)^{\frac{k-1}{k}}\right]} + (Pe - Pa) \cdot Ae$$

Where :
k = Ratio of specific heats
Pc = Combustion chamber pressure

Dividing both sides by Pc*Astar yields:

$$\frac{Thrust}{Pc \cdot Astar} := \sqrt{\frac{2 \cdot k^2}{k-1}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}} \cdot \left[1 - \left(\frac{Pe}{Pc}\right)^{\frac{k-1}{k}}\right]} + \frac{(Pe - Pa) \cdot Ae}{Pc \cdot Astar}$$

The expression on the right hand side of the equal sign is known as the **Thrust Coefficient** (Cf).

$$Cf := \sqrt{\frac{2\cdot k^2}{k-1}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}\cdot\left[1-\left(\frac{Pe}{Pc}\right)^{\frac{k-1}{k}}\right]}+\frac{(Pe-Pa)\cdot Ae}{Pc\cdot Astar}$$

Therefore :

$$\frac{Thrust}{Pc\cdot Astar} := Cf$$

Since force is Pressure * Area, the thrust coefficient can be thought of as a measure of amount of thrust amplification the nozzle produces.

$$Thrust := (Pc\cdot Astar)\cdot Cf$$

In this particular case we would like to solve for Astar (A*) to find the throat area we need for our nozzle design.

$$Astar := \frac{Thrust}{Pc\cdot Cf}$$

Therefore :

$$Astar := \frac{Thrust}{Pc\cdot\left[\sqrt{\frac{2\cdot k^2}{k-1}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}\cdot\left[1-\left(\frac{Pe}{Pc}\right)^{\frac{k-1}{k}}\right]}+\frac{(Pe-Pa)\cdot Ae}{Pc\cdot Astar}\right]}$$

We see that Astar now appears on both side so the equal sign. We have a relationship that will remove it from the right side of the equation.

Once again using the magical equations for:

1. Energy
2. Continuity
3. Momentum
4. Equation of state

We achieve the following equation: This section to be expanded upon

$$\frac{Astar}{Ae} := \left(\frac{k+1}{2}\right)^{\frac{1}{k-1}}\cdot\left(\frac{Pe}{Pc}\right)^{\frac{1}{k}}\cdot\sqrt{\frac{k+1}{k-1}\cdot\left[1-\left(\frac{Pe}{Pc}\right)^{\frac{k-1}{k}}\right]}$$

or inverting it yields:

$$\frac{Ae}{Astar} := \frac{1}{\left[\left(\frac{k+1}{2}\right)^{\frac{1}{k-1}}\cdot\left(\frac{Pe}{Pc}\right)^{\frac{1}{k}}\cdot\sqrt{\frac{k+1}{k-1}\cdot\left[1-\left(\frac{Pe}{Pc}\right)^{\frac{k-1}{k}}\right]}\right]}$$

This is know as the **optimal expansion ratio** for the nozzle. This is our target design condition to achieve maximum thrust.

We can substitute this equation into the previous equation to get rid of the Astar term on the right hand side of the expression. Doing this yields:

$$\mathrm{Astar} := \frac{\mathrm{Thrust}}{\mathrm{Pc}\cdot\mathrm{Cf}}$$

$$\mathrm{Astar} := \frac{\mathrm{Thrust}}{\mathrm{Pc}\cdot\left[\sqrt{\frac{2\cdot k^2}{k-1}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}\cdot\left[1-\left(\frac{\mathrm{Pe}}{\mathrm{Pc}}\right)^{\frac{k-1}{k}}\right]}+\frac{(\mathrm{Pe}-\mathrm{Pa})}{\mathrm{Pc}}\cdot\frac{1}{\left[\left(\frac{k+1}{2}\right)^{\frac{1}{k-1}}\cdot\left(\frac{\mathrm{Pe}}{\mathrm{Pc}}\right)^{\frac{1}{k}}\cdot\sqrt{\frac{k+1}{k-1}\cdot\left[1-\left(\frac{\mathrm{Pe}}{\mathrm{Pc}}\right)^{\frac{k-1}{k}}\right]}\right]}\right]}$$

This monster of an equation now is able to solve for Astar (the design target nozzle throat area) in terms of the following variables:

Thrust = Motor output force
k = Ratio of specific heats
Pe = Exit pressure of nozzle
Pc = Combustion chamber pressure
Pa = Ambient pressure

Calculations:

For our static test firing of the gaseous oxygen/ paraffin hybrid motor we know the following:

1) Our design requirements specify the motor has and output thrust of 50 lbf.

$$\mathrm{Thrust} := 50\mathrm{lbf}$$

2) The ratio of specific heats for an oxygen and paraffin propellant mixture is approx. 1.2. This number was generated by PEP and is the average of the k value of the gases in combustion chamber and the k value for the gases in the exhaust stream. See **appendix A** for details:

$$k := 1.2$$

3) We will assume that the pressure at the nozzle exit (Pe) is equal to the ambient pressure (Pa) which is equal to 14.6 psi.

$$\mathrm{Pa} := 14.7\mathrm{psi}$$

$$\mathrm{Pe} := \mathrm{Pa}$$

4) Our design requirements specify motor chamber pressure to be 500 psi.

$$\mathrm{Pc} := 500\mathrm{psi}$$

We now have enough information to solve for our required throat area. Substituting ; Thrust, k, Pa, Pe, and Pc into the equation for Astar yields:

$$Astar := \frac{Thrust}{Pc\cdot\left[\sqrt{\frac{2\cdot k^2}{k-1}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}\cdot\left[1-\left(\frac{Pe}{Pc}\right)^{\frac{k-1}{k}}\right]}+\frac{(Pe-Pa)}{Pc}\cdot\frac{1}{\left[\left(\frac{k+1}{2}\right)^{\frac{1}{k-1}}\cdot\left(\frac{Pe}{Pc}\right)^{\frac{1}{k}}\cdot\sqrt{\frac{k+1}{k-1}\cdot\left[1-\left(\frac{Pe}{Pc}\right)^{\frac{k-1}{k}}\right]}\right]}\right]}$$

$$Astar = 0.067\,in^2$$

In our particular situation of static firing the motor we will target our exit pressure to equal that of the ambient pressure Pe = Pa the term in the denominator simplifies to :

$$Astar := \frac{Thrust}{Pc\cdot\left[\sqrt{\frac{2\cdot k^2}{k-1}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}\cdot\left[1-\left(\frac{Pe}{Pc}\right)^{\frac{k-1}{k}}\right]}\right]}$$

$$Astar = 0.067\,in^2$$

We can calculate the required nozzle throat diameter from:

$$Astar := \frac{\pi\cdot Dthroat^2}{4}$$

or

$$Dthroat := \sqrt{\frac{4\cdot Astar}{\pi}}$$

$$Dthroat = 0.292\,in$$

We can now simply solve for our nozzle exit area by using our formula for optimal expansion ratio combined with the nozzle throat area we have just determined.

$$Ae := Astar\cdot\frac{Ae}{Astar}$$

$$Ae := Astar\cdot\frac{1}{\left[\left(\frac{k+1}{2}\right)^{\frac{1}{k-1}}\cdot\left(\frac{Pe}{Pc}\right)^{\frac{1}{k}}\cdot\sqrt{\frac{k+1}{k-1}\cdot\left[1-\left(\frac{Pe}{Pc}\right)^{\frac{k-1}{k}}\right]}\right]}$$

Therefore

$$Ae = 0.354\,in^2$$

We can calculate the required nozzle throat diameter from:

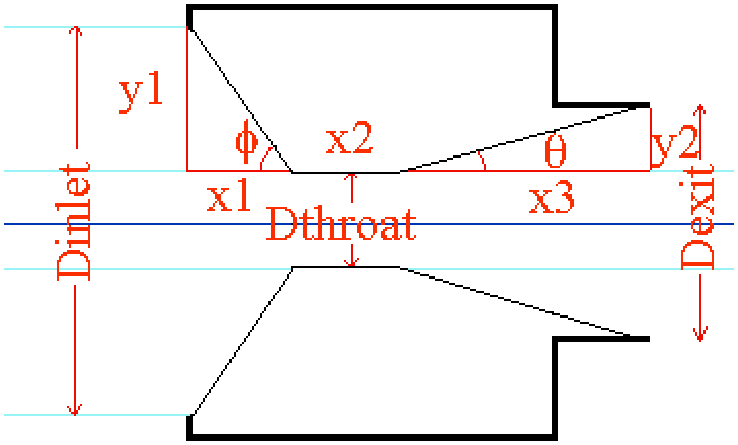
$$Ae := \frac{\pi \cdot Dexit^2}{4}$$

or

$$Dexit := \sqrt{\frac{4 \cdot Ae}{\pi}}$$

$$Dexit = 0.672\,in$$

We are now ready to determine the other dimensions for that nozzle that will fully define the geometry so that we may fabricate a test article:



We will set the following parameters based on what has worked well for other amateur motor builders in the past.

$$\theta := 17deg$$

$$\phi := 60deg$$

We arbitrarily will set the nozzle throat thickness (x2) to 0.25inches. This will hopefully minimize any effects of erosion to the graphite during static firing.

$$x2 := 0.5\text{in}$$

We would now like to find the required depth of the converging section of the nozzle (x1):

From our prototype design specifications we know Dinlet to be 1.75 inches.

$$\text{Dinlet} := 1.75\text{in}$$

Therefore we know that:

$$y1 := \frac{\text{Dinlet} - \text{Dthroat}}{2}$$

$$y1 = 0.729\,\text{in}$$

and we know from the trigonometric relation:

$$\tan(\phi) := \frac{y1}{x1}$$

or

$$x1 := \frac{y1}{\tan(\phi)}$$

$$x1 = 0.421\,\text{in}$$

We would now like to find the required depth of the diverging section of the nozzle (x3)

Therefore we know that:

$$y3 := \frac{\text{Dexit} - \text{Dthroat}}{2}$$

$$y3 = 0.191\,\text{in}$$

and we know from the trigonometric relation:

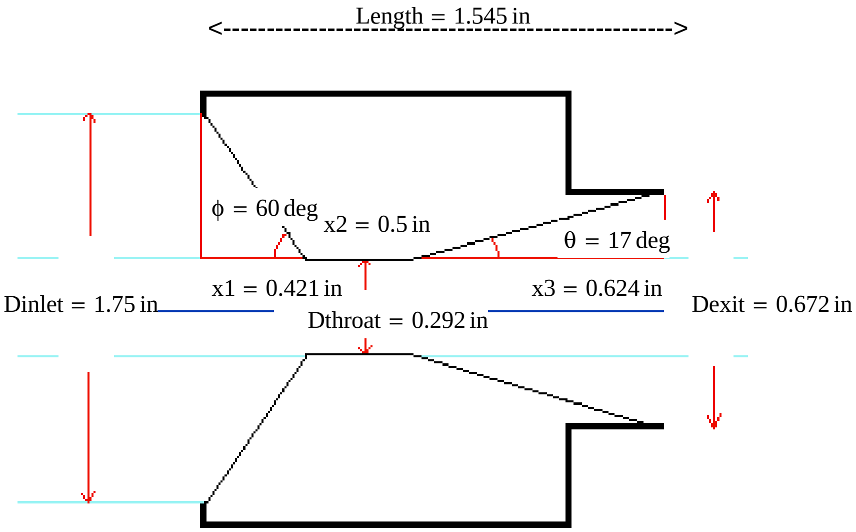
$$\tan(\phi) := \frac{y3}{x3}$$

or

$$x3 := \frac{y3}{\tan(\theta)}$$

$$x3 = 0.624\,\text{in}$$

The design of the hybrid nozzle is now fully defined.



Since our ratio of specific heats was calculated using a software package that assumes ideal conditions we would like to do a check for sensitivity of throat diameter and optimal expansion ratio to small changes in k. The following spread sheet shows the results.

Sensitivity of Ae/A* and Dthroat to k					
Given: Pc=500 psi					
Pe=Pa=14.7 psi					
Ratio of specific heats (k)		Expansion ratio (Ae/A*)	Throat areat (A*) in^2	Throat diameter (Dthroat) inches	
1.01		7.60	0.062	0.282	
1.10		6.31	0.065	0.287	
1.20		5.31	0.067	0.292	
1.30		4.58	0.068	0.295	
1.40		4.03	0.069	0.297	
1.50		3.61	0.070	0.299	

For a 0.50 change in k which is fairly significant the nozzle throat diameter changes only 0.017 inches. Leading us to the conclusion that the a nozzle throat diameter of 0.29 inches will be a good first choice even if we are not precisely sure what our k value for the GOX/Paraffin mixture is.

Footnotes :

[1] http://members.aol.com/ricnakk/

[2] "The Design, Construction and Testing of a Liquid Propellant Rocket", Benito Loyola Midshipman 1/c 1988. GDM & Co., Santa Paula, CA. Distributed by Pacific Rocket Society

[3] "Thermodynamics, An Engineering Approach" 4th Ed., Yunus A Cengel, Michael A Boles. 2002, McGraw-Hill

[4] "Fundamentals of Fluid Mechanics" 4th Ed., Bruce R. Munson, Donald F. Young, Theodore K Okiishi. 2002 John Wiley & Sons, Inc.

[5] NWC Propellant Evaluation Program
http://lekstutis.com/Artie/PEP/

Appendix A

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 GOXHybrid        Run using June 1988 Version of PEP,
 Case  1 of  1    14 Nov 2003 at  8: 4:56.72 pm

  CODE                                 WEIGHT   D-H  DENS     COMPOSITION
   736 OXYGEN (GAS)                   648.000     0    0.00001        2O
  1098 EICOSANE (PARAFFIN)            240.000  -469    0.00001        20C  42H

THE PROPELLANT DENSITY IS 0.00001 LB/CU-IN OR  0.0003 GM/CC
THE TOTAL PROPELLANT WEIGHT IS  888.0000 GRAMS

NUMBER OF GRAM ATOMS OF EACH ELEMENT PRESENT IN INGREDIENTS

  35.674343 H    16.987783 C    40.500000 O

****************************CHAMBER RESULTS FOLLOW    *****************************

 T(K)  T(F)  P(ATM)   P(PSI) ENTHALPY  ENTROPY      CP/CV     GAS     RT/V
3610. 6039.   34.01   500.00  -112.56     2474.92       1.2166    38.729   0.878

SPECIFIC HEAT (MOLAR) OF GAS AND TOTAL=   11.156   11.160
NUMBER MOLS GAS AND CONDENSED=  38.7285   0.0000

 12.45997 H2O      11.58498 CO       5.40181 CO2       3.25228 H2
  2.79286 HO        1.45048 H         1.06695 O2        0.71277 O
 5.55E-03 HO2      6.69E-04 CHO      1.84E-05 CH2O

THE MOLECULAR WEIGHT OF THE MIXTURE IS   22.929
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****************************EXHAUST RESULTS FOLLOW    *****************************

 T(K)  T(F)  P(ATM)   P(PSI) ENTHALPY  ENTROPY        CP/CV    GAS      RT/V
2655. 4319.    1.00    14.70  -922.01        2474.92           1.2027      35.611    0.028

SPECIFIC HEAT (MOLAR) OF GAS AND TOTAL=   11.789   11.790
NUMBER MOLS GAS AND CONDENSED=  35.6109   0.0000

 14.65390 H2O       9.16824 CO        7.81922 CO2       2.63041 H2
  0.64330 HO        0.46202 H         0.16220 O2        0.07132 O
 1.38E-04 HO2       1.34E-05 CHO

THE MOLECULAR WEIGHT OF THE MIXTURE IS   24.936

**********PERFORMANCE:  FROZEN ON FIRST LINE, SHIFTING ON SECOND LINE**********

IMPULSE   IS EX      T*      P*      C*    ISP*  OPT-EX  D-ISP      A*M   EX-T
  266.2  1.2271   3242.   19.02  5729.6           5.09    0.1  0.35624  1880.
  281.7  1.1274   3444.   19.69  5923.8   226.7    6.04    0.1  0.36832  2655.


OOST VELOCITIES FOR PROPELLANT DENSITY OF  0.00001  (S.G. OF    0.000)

  5./   31.  10./   16.  15./   10.  25./    6.  30./    5.  55./    3.
 60./    3.  69./    2.  71./    2.  88./    2. 100./    2. 150./    1.
 175./   1.  200./   1.  300./   1. 1000./   0. 3000./   0. 5000./   0.

 EXP.   EXIT   EXIT   EXIT OPTIMUM OPTIMUM VACUUM VACUUM SEA LV SEA LV
RATIO  PRESS  PRESS   TEMP IMPULSE IMPULSE IMPULS IMPULS IMPULS IMPULS
        ATM     SI      K     SEC      SI    SEC     SI    SEC     SI
   1. 19.689 1994.5  3444.   120.1   1178.  226.7  2223.  221.3  2192.
   2.  6.563  664.8  3132.   202.1   1982.  273.2  2679.  262.3  2599.
   3.  2.534  256.7  2880.   247.6   2428.  288.8  2832.  272.5  2700.
   4.  1.716  173.8  2783.   262.9   2578.  300.0  2942.  278.4  2757.
   5.  1.278  129.4  2712.   273.4   2681.  308.0  3020.  280.9  2783.
   6.  1.008  102.1  2656.   281.4   2760.  314.1  3081.  281.7  2790.
   7.  0.827   83.8  2611.   287.8   2822.  319.1  3129.  281.2  2786.
   8.  0.697   70.6  2572.   293.0   2873.  323.2  3169.  279.9  2773.
   9.  0.601   60.9  2538.   297.4   2917.  326.7  3204.  278.0  2754.
  10.  0.526   53.3  2509.   301.3   2954.  329.8  3234.  275.6  2730.
  11.  0.467   47.3  2483.   304.6   2987.  332.4  3260.  272.9  2703.
  12.  0.419   42.4  2459.   307.6   3017.  334.8  3283.  269.9  2673.
  13.  0.379   38.4  2438.   310.3   3043.  337.0  3305.  266.6  2641.
  14.  0.346   35.0  2418.   312.7   3067.  339.0  3324.  263.2  2607.
  15.  0.318   32.2  2400.   315.0   3089.  340.7  3341.  259.6  2571.
  16.  0.293   29.7  2383.   317.0   3109.  342.4  3358.  255.8  2534.
  17.  0.272   27.6  2368.   318.9   3127.  343.9  3373.  251.9  2495.
  18.  0.254   25.7  2353.   320.6   3144.  345.4  3387.  247.9  2456.
  19.  0.237   24.0  2340.   322.3   3160.  346.7  3400.  243.8  2415.
  20.  0.223   22.6  2327.   323.8   3175.  347.9  3412.  239.7  2374.

  21   0.210   21.3  2314    325.2   3189   349.1  3423   235.4  2332
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  21.  0.210   21.3  2314.   325.2   3189.  349.1  3423.  235.4  2332.
  22.  0.198   20.1  2303.   326.6   3203.  350.2  3434.  231.1  2289.
  23.  0.188   19.0  2292.   327.9   3215.  351.3  3444.  226.7  2246.
  24.  0.178   18.1  2282.   329.1   3227.  352.2  3454.  222.3  2202.
  25.  0.170   17.2  2272.   330.2   3238.  353.2  3463.  217.9  2158.


 EXP.   EXIT   EXIT   EXIT OPTIMUM OPTIMUM VACUUM VACUUM SEA LV SEA LV
RATIO  PRESS  PRESS   TEMP IMPULSE IMPULSE IMPULS IMPULS IMPULS IMPULS
        ATM     SI     K     SEC      SI    SEC     SI    SEC     SI
  26.  0.162   16.4  2262.   331.3   3249.  354.1  3472.  213.3  2113.
  27.  0.155   15.7  2253.   332.4   3259.  354.9  3481.  208.8  2068.
  28.  0.148   15.0  2244.   333.4   3269.  355.8  3489.  204.2  2023.
  29.  0.142   14.4  2236.   334.3   3278.  356.5  3496.  199.6  1977.
  30.  0.136   13.8  2228.   335.2   3287.  357.3  3504.  194.9  1931.
  31.  0.131   13.2  2220.   336.1   3296.  358.0  3511.  190.2  1884.
  32.  0.126   12.7  2213.   336.9   3304.  358.7  3518.  185.5  1837.
  33.  0.121   12.3  2205.   337.7   3312.  359.4  3524.  180.7  1790.
  34.  0.117   11.8  2198.   338.5   3320.  360.0  3531.  176.0  1743.
  35.  0.113   11.4  2192.   339.3   3327.  360.7  3537.  171.2  1696.
  36.  0.109   11.0  2185.   340.0   3334.  361.3  3543.  166.4  1648.
  37.  0.105   10.7  2179.   340.7   3341.  361.8  3548.  161.5  1600.
  38.  0.102   10.3  2173.   341.4   3348.  362.4  3554.  156.7  1552.
  39.  0.099   10.0  2167.   342.0   3354.  362.9  3559.  151.8  1504.
  40.  0.096    9.7  2161.   342.7   3360.  363.5  3564.  146.9  1456.
  41.  0.093    9.4  2155.   343.3   3366.  364.0  3569.  142.0  1407.
  42.  0.091    9.2  2150.   343.9   3372.  364.5  3574.  137.1  1358.
  43.  0.088    8.9  2144.   344.5   3378.  365.0  3579.  132.2  1310.
  44.  0.086    8.7  2139.   345.0   3384.  365.4  3584.  127.2  1261.
  45.  0.083    8.4  2134.   345.6   3389.  365.9  3588.  122.3  1211.
  46.  0.081    8.2  2129.   346.1   3394.  366.3  3592.  117.3  1162.
  47.  0.079    8.0  2124.   346.6   3399.  366.8  3597.  112.3  1113.
  48.  0.077    7.8  2120.   347.2   3404.  367.2  3601.  107.4  1063.
  49.  0.075    7.6  2115.   347.6   3409.  367.6  3605.  102.4  1014.
  50.  0.073    7.4  2111.   348.1   3414.  368.0  3609.   97.3   964.
  51.  0.072    7.3  2106.   348.6   3418.  368.4  3613.   92.3   915.
  52.  0.070    7.1  2102.   349.1   3423.  368.8  3616.   87.3   865.
  53.  0.068    6.9  2098.   349.5   3427.  369.2  3620.   82.3   815.
  54.  0.067    6.8  2093.   350.0   3432.  369.5  3624.   77.2   765.
  55.  0.066    6.6  2089.   350.4   3436.  369.9  3627.   72.2   715.
  56.  0.064    6.5  2085.   350.8   3440.  370.2  3631.   67.1   665.
  57.  0.063    6.4  2082.   351.2   3444.  370.6  3634.   62.0   614.
  58.  0.061    6.2  2078.   351.6   3448.  370.9  3637.   56.9   564.
  59.  0.060    6.1  2074.   352.0   3452.  371.2  3641.   51.9   514.
  60.  0.059    6.0  2070.   352.4   3456.  371.6  3644.   46.8   463.
  61.  0.058    5.9  2067.   352.8   3459.  371.9  3647.   41.7   413.
  62.  0.057    5.7  2063.   353.1   3463.  372.2  3650.   36.6   362.
  63.  0.056    5.6  2060.   353.5   3467.  372.5  3653.   31.5   312.
  64.  0.055    5.5  2056.   353.9   3470.  372.8  3656.   26.4   261.
  65.  0.054    5.4  2053.   354.2   3474.  373.1  3659.   21.2   210.
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 EXP.   EXIT   EXIT   EXIT OPTIMUM OPTIMUM VACUUM VACUUM SEA LV SEA LV
RATIO  PRESS  PRESS   TEMP IMPULSE IMPULSE IMPULS IMPULS IMPULS IMPULS
        ATM     SI      K     SEC      SI    SEC     SI    SEC     SI
  66.  0.053    5.3  2050.   354.6   3477.  373.4  3661.   16.1   160.
  67.  0.052    5.2  2047.   354.9   3480.  373.7  3664.   11.0   109.
  68.  0.051    5.1  2043.   355.2   3483.  373.9  3667.    5.8    58.
  69.  0.050    5.1  2040.   355.6   3487.  374.2  3670.    0.7     7.
  70.  0.049    5.0  2037.   355.9   3490.  374.5  3672.   -4.4   -44.
  71.  0.048    4.9  2034.   356.2   3493.  374.8  3675.   -9.6   -95.
  72.  0.047    4.8  2031.   356.5   3496.  375.0  3677.  -14.7  -146.
  73.  0.047    4.7  2028.   356.8   3499.  375.3  3680.  -19.9  -197.
  74.  0.046    4.7  2025.   357.1   3502.  375.5  3682.  -25.1  -248.
  75.  0.045    4.6  2023.   357.4   3505.  375.8  3685.  -30.2  -300.
  76.  0.045    4.5  2020.   357.7   3508.  376.0  3687.  -35.4  -351.
  77.  0.044    4.4  2017.   358.0   3510.  376.2  3690.  -40.6  -402.
  78.  0.043    4.4  2014.   358.3   3513.  376.5  3692.  -45.8  -453.
  79.  0.043    4.3  2012.   358.5   3516.  376.7  3694.  -50.9  -505.
  80.  0.042    4.2  2009.   358.8   3519.  376.9  3696.  -56.1  -556.
  81.  0.041    4.2  2006.   359.1   3521.  377.2  3699.  -61.3  -607.
  82.  0.041    4.1  2004.   359.3   3524.  377.4  3701.  -66.5  -659.
  83.  0.040    4.1  2001.   359.6   3526.  377.6  3703.  -71.7  -710.
  84.  0.040    4.0  1999.   359.9   3529.  377.8  3705.  -76.9  -762.
  85.  0.039    3.9  1996.   360.1   3531.  378.0  3707.  -82.1  -813.
  86.  0.038    3.9  1994.   360.4   3534.  378.2  3709.  -87.3  -865.
  87.  0.038    3.8  1991.   360.6   3536.  378.5  3711.  -92.5  -916.
  88.  0.037    3.8  1989.   360.8   3539.  378.7  3713.  -97.7  -968.
  89.  0.037    3.7  1987.   361.1   3541.  378.9  3715. -102.9 -1020.
  90.  0.036    3.7  1984.   361.3   3543.  379.1  3717. -108.1 -1071.
  91.  0.036    3.6  1982.   361.6   3545.  379.3  3719. -113.4 -1123.
  92.  0.035    3.6  1980.   361.8   3548.  379.4  3721. -118.6 -1175.
  93.  0.035    3.5  1978.   362.0   3550.  379.6  3723. -123.8 -1226.
  94.  0.035    3.5  1975.   362.2   3552.  379.8  3725. -129.0 -1278.
  95.  0.034    3.5  1973.   362.5   3554.  380.0  3726. -134.3 -1330.
  96.  0.034    3.4  1971.   362.7   3556.  380.2  3728. -139.5 -1382.
  97.  0.033    3.4  1969.   362.9   3559.  380.4  3730. -144.7 -1434.
  98.  0.033    3.3  1967.   363.1   3561.  380.6  3732. -149.9 -1485.
  99.  0.033    3.3  1965.   363.3   3563.  380.7  3734. -155.2 -1537.
 100.  0.032    3.3  1963.   363.5   3565.  380.9  3735. -160.4 -1589.
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